How Arrow Switching can provide Fair Competition

This note seeks to explain, with the aid of examples, how 'competition' between pairs is measured, how it is affected by the choice of movement, how the 'fairness' of a single-winner movement can be assessed – and therefore when it is and is not reasonable to arrow switch.

Single-winner Mitchell movements: Switching

In an unswitched Mitchell, each pair is compared with slightly less than half the field, namely just the other pairs sitting in the same direction. If there are 7 tables, for example, each pair is compared with only 6 of the 13 other pairs. However, the comparisons can (under certain conditions) be 'perfect'.

Switching enables each pair to be compared with all the other pairs. With the correct amount of switching, the switched Mitchell still provides a high degree of fairness, though it cannot be 'perfectly' fair.

8 Tables

Different Mitchell movements are available, including Share-&-Relay, Skip, and Double Weave.

8 Round Share-&-Relay Mitchell

The following table shows the amounts of 'competition' between pairs. (The Double Weave Mitchell produces an identical table.)

		NS								EW							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
NS	**1**	0	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0
	2	8	0	8	8	8	8	8	8	0	0	0	0	0	0	0	0
	3	8	8	0	8	8	8	8	8	0	0	0	0	0	0	0	0
	4	8	8	8	0	8	8	8	8	0	0	0	0	0	0	0	0
	5	8	8	8	8	0	8	8	8	0	0	0	0	0	0	0	0
	6	8	8	8	8	8	0	8	8	0	0	0	0	0	0	0	0
	7	8	8	8	8	8	8	0	8	0	0	0	0	0	0	0	0
	8	8	8	8	8	8	8	8	0	0	0	0	0	0	0	0	0
EW	**9**	0	0	0	0	0	0	0	0	0	8	8	8	8	8	8	8
	10	0	0	0	0	0	0	0	0	8	0	8	8	8	8	8	8
	11	0	0	0	0	0	0	0	0	8	8	0	8	8	8	8	8
	12	0	0	0	0	0	0	0	0	8	8	8	0	8	8	8	8
	13	0	0	0	0	0	0	0	0	8	8	8	8	0	8	8	8
	14	0	0	0	0	0	0	0	0	8	8	8	8	8	0	8	8
	15	0	0	0	0	0	0	0	0	8	8	8	8	8	8	0	8
	16	0	0	0	0	0	0	0	0	8	8	8	8	8	8	8	0

The entries in the table are derived as follows.

Each NS is compared with every other NS over 8 rounds. Between any two NS pairs, 1MP (match point) can be won or lost on each board (by getting a better or a worse result than the other NS). So over 8 rounds (sets of boards) the total competition between any two NS pairs is 8MP.[1] Similarly each EW is compared with every other EW over 8 rounds; and the total competition between any two EW pairs is 8MP.

When they play each other, a NS and an EW compete for all the available match points: there is 'half a top', which here is 7MP, to be won or lost (by getting a better or worse result than all the 7 other pairs playing the board in the same direction). However, over the remaining 7 rounds of the session they *cooperate* for 1MP on each board. Hence the total *net* competition between any NS and any EW is 7 - 7x1 = 0MP.

The movement therefore effectively delivers two entirely separate competitions, one among NSs and the other among EWs.

[1] times the number of boards per set. For simplicity, it is assumed that there is just one board per set. If there were 3 boards per set then all the entries would be multiplied by 3.

Effect of a Half Table

With a phantom NS at Table 8, we now have

		NS								EW							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
NS	**1**	0	8	8	8	8	8	8	0	-1	-1	-1	-1	-1	-1	-1	-1
	2	8	0	8	8	8	8	8	0	-1	-1	-1	-1	-1	-1	-1	-1
	3	8	8	0	8	8	8	8	0	-1	-1	-1	-1	-1	-1	-1	-1
	4	8	8	8	0	8	8	8	0	-1	-1	-1	-1	-1	-1	-1	-1
	5	8	8	8	8	0	8	8	0	-1	-1	-1	-1	-1	-1	-1	-1
	6	8	8	8	8	8	0	8	0	-1	-1	-1	-1	-1	-1	-1	-1
	7	8	8	8	8	8	8	0	0	-1	-1	-1	-1	-1	-1	-1	-1
	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EW	**9**	-1	-1	-1	-1	-1	-1	-1	0	0	8	8	8	8	8	8	8
	10	-1	-1	-1	-1	-1	-1	-1	0	8	0	8	8	8	8	8	8
	11	-1	-1	-1	-1	-1	-1	-1	0	8	8	0	8	8	8	8	8
	12	-1	-1	-1	-1	-1	-1	-1	0	8	8	8	0	8	8	8	8
	13	-1	-1	-1	-1	-1	-1	-1	0	8	8	8	8	0	8	8	8
	14	-1	-1	-1	-1	-1	-1	-1	0	8	8	8	8	8	0	8	8
	15	-1	-1	-1	-1	-1	-1	-1	0	8	8	8	8	8	8	0	8
	16	-1	-1	-1	-1	-1	-1	-1	0	8	8	8	8	8	8	8	0

Evidently there is no competition between the phantom NS (pair 8) and any other pair; but all the other NSs compete with each other for 1MP per board over 8 rounds as before; and similarly for EWs.

However each board is now played one time fewer; so when a NS plays an EW only 6MP are now at stake (only 6 other pairs play the board in the same direction). But they still cooperate for 1MP on each board for the remaining 7 rounds. Hence the cooperation now outweighs the competition between them, and the total *net* competition between any NS and any EW is -1MP.

8 Round Skip Mitchell with a 'Revenge' round

		NS								EW							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
NS	**1**	0	8	8	8	8	8	8	8	8	0	0	0	-8	0	0	0
	2	8	0	8	8	8	8	8	8	0	8	0	0	0	-8	0	0
	3	8	8	0	8	8	8	8	8	0	0	8	0	0	0	-8	0
	4	8	8	8	0	8	8	8	8	0	0	0	8	0	0	0	-8
	5	8	8	8	8	0	8	8	8	-8	0	0	0	8	0	0	0
	6	8	8	8	8	8	0	8	8	0	-8	0	0	0	8	0	0
	7	8	8	8	8	8	8	0	8	0	0	-8	0	0	0	8	0
	8	8	8	8	8	8	8	8	0	0	0	0	-8	0	0	0	8
EW	**9**	8	0	0	0	-8	0	0	0	0	8	8	8	8	8	8	8
	10	0	8	0	0	0	-8	0	0	8	0	8	8	8	8	8	8
	11	0	0	8	0	0	0	-8	0	8	8	0	8	8	8	8	8
	12	0	0	0	8	0	0	0	-8	8	8	8	0	8	8	8	8
	13	-8	0	0	0	8	0	0	0	8	8	8	8	0	8	8	8
	14	0	-8	0	0	0	8	0	0	8	8	8	8	8	0	8	8
	15	0	0	-8	0	0	0	8	0	8	8	8	8	8	8	0	8
	16	0	0	0	-8	0	0	0	8	8	8	8	8	8	8	8	0

The competition between any two NS pairs (and between any two EW pairs) is the same as before. Between a NS and an EW the amount of competition can also be zero, as before, but only if they meet just once. If they do not meet at all (the EW skips that NS, as 13 skips 1) there is no direct competition at all to offset the 8 rounds of cooperation, and the *net* competition is -8MP. If they meet twice (start at the same table and play a 'revenge' round, as do 1 & 9) then there is an extra round of direct competition (for 7MP) and correspondingly one fewer round of cooperation (1MP). The overall effect is to make the *net* competition 8MP per board, which is as strong as the competition between pairs sitting in the same direction.

For this reason it is much less appropriate to regard the movement as delivering two entirely separate competitions, one among NSs and the other among EWs. However, *as a single-winner movement* it is highly 'unfair'. A perfectly fair movement has equal competition between all pairs and its 'balance' is 100%. Here the amount of competition between pairs varies greatly and the balance of this movement is only 36%.[i]

Effect of Arrow Switching the 8 Round Skip Mitchell with a 'Revenge' round

		Static (NS)								Moving (EW)							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Static (NS)	**1**	0	4	4	4	4	4	4	4	8	4	4	4	-4	4	4	4
	2	4	0	4	4	4	4	4	4	4	8	4	4	4	-4	4	4
	3	4	4	0	4	4	4	4	4	4	4	8	4	4	4	-4	4
	4	4	4	4	0	4	4	4	4	4	4	4	8	4	4	4	-4
	5	4	4	4	4	0	4	4	4	-4	4	4	4	8	4	4	4
	6	4	4	4	4	4	0	4	4	4	-4	4	4	4	8	4	4
	7	4	4	4	4	4	4	0	4	4	4	-4	4	4	4	8	4
	8	4	4	4	4	4	4	4	0	4	4	4	-4	4	4	4	8
Moving (EW)	**9**	8	4	4	4	-4	4	4	4	0	4	4	4	4	4	4	4
	10	4	8	4	4	4	-4	4	4	4	0	4	4	4	4	4	4
	11	4	4	8	4	4	4	-4	4	4	4	0	4	4	4	4	4
	12	4	4	4	8	4	4	4	-4	4	4	4	0	4	4	4	4
	13	-4	4	4	4	8	4	4	4	4	4	4	4	0	4	4	4
	14	4	-4	4	4	4	8	4	4	4	4	4	4	4	0	4	4
	15	4	4	-4	4	4	4	8	4	4	4	4	4	4	4	0	4
	16	4	4	4	-4	4	4	4	8	4	4	4	4	4	4	4	0

The effect of switching just one round (the 'revenge' round) is reflected in the table above.

- Between any two static pairs, net competition is *reduced* by 4. The static pair which plays the switched round as EW now cooperates rather than competes with each of the static pairs which play it as NS; and on each of the other rounds, which it plays as NS, it cooperates rather than competes with the static pair which plays it arrow switched. Hence it competes with each other static pair two times fewer and correspondingly cooperates with it two times more; and so the *net* competition between them is reduced by 4 (from 8 to 4). Similarly the net competition between moving pairs is now also 4.
- Between a static pair and a moving pair which play each other only on *un*switched rounds, the net competition is *increased* by 4. The static pair which plays the switched round as EW now competes rather than cooperates with each of the moving pairs which play it as EW; and on each of the other rounds, which it plays as NS, it competes rather than cooperates with the moving pair which plays it arrow switched. Hence it competes with each other moving pair two times more and correspondingly cooperates with it two times fewer; and so the *net* competition between them is increased by 4 (from 0 to 4 or from -8 to -4).
- Between a static pair and a moving pair which play each other on a switched round (here, 1 & 9, 2 & 10, etc), nothing changes. Because of the revenge round, they compete twice head-to-head for 7MP each time, and on the other 6 rounds cooperate for 1MP, and the *net* competition between them remains 8.

As a result of switching one round, there is now equal competition between many pairs (but far from all); the balance is now 73%.

Arrow Switching an 8 Round Double Weave Mitchell

		Static (NS)								Moving (EW)							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Static (NS)	**1**	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0
	2	4	0	4	4	4	4	4	4	4	4	0	4	4	4	4	4
	3	4	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4
	4	4	4	4	0	4	4	4	4	4	4	4	4	0	4	4	4
	5	4	4	4	4	0	4	4	4	4	4	4	0	4	4	4	4
	6	4	4	4	4	4	0	4	4	4	4	4	4	4	4	0	4
	7	4	4	4	4	4	4	0	4	4	4	4	4	4	0	4	4
	8	4	4	4	4	4	4	4	0	0	4	4	4	4	4	4	4
Moving (EW)	**9**	4	4	4	4	4	4	4	0	0	4	4	4	4	4	4	4
	10	4	4	0	4	4	4	4	4	4	0	4	4	4	4	4	4
	11	4	0	4	4	4	4	4	4	4	4	0	4	4	4	4	4
	12	4	4	4	4	0	4	4	4	4	4	4	0	4	4	4	4
	13	4	4	4	0	4	4	4	4	4	4	4	4	0	4	4	4
	14	4	4	4	4	4	4	0	4	4	4	4	4	4	0	4	4
	15	4	4	4	4	4	0	4	4	4	4	4	4	4	4	0	4
	16	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	0

The effects on net competition are the same as in the previous case: +4, -4, or 0. However, the Double Weave Mitchell was better balanced than the Skip Mitchell, and switching it produces a correspondingly better-balanced competition array. The balance is now 93%. (An arrow switched Share-&-Relay Mitchell is as good.)

Effect of too much switching

In the previous examples, one round in eight was switched, so that one eighth of the boards were switched. If the last two rounds of a Double Weave are switched (so that one quarter of the boards are switched), the table looks as follows.

		Static (NS)								Moving (EW)							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Static (NS)	**1**	0	0	0	0	0	0	0	8	4	8	4	8	8	4	8	4
	2	0	0	8	0	0	0	0	0	8	4	4	8	4	8	8	4
	3	0	8	0	0	0	0	0	0	8	4	4	8	4	8	8	4
	4	0	0	0	0	8	0	0	0	8	4	8	4	4	8	4	8
	5	0	0	0	8	0	0	0	0	8	4	8	4	4	8	4	8
	6	0	0	0	0	0	0	8	0	4	8	8	4	8	4	4	8
	7	0	0	0	0	0	8	0	0	4	8	8	4	8	4	4	8
	8	8	0	0	0	0	0	0	0	4	8	4	8	8	4	8	4
Moving (EW)	**9**	4	8	8	8	8	4	4	4	0	0	4	0	0	0	4	0
	10	8	4	4	4	4	8	8	8	0	0	0	4	0	0	0	4
	11	4	4	4	8	8	8	8	4	4	0	0	0	4	0	0	0
	12	8	8	8	4	4	4	4	8	0	4	0	0	0	4	0	0
	13	8	4	4	4	4	8	8	8	0	0	4	0	0	0	4	0
	14	4	8	8	8	8	4	4	4	0	0	0	4	0	0	0	4
	15	8	8	8	4	4	4	4	8	4	0	0	0	4	0	0	0
	16	4	4	4	8	8	8	8	4	0	4	0	0	0	4	0	0

The balance falls to 57%.

This confirms the theoretical result that it is best to switch (roughly) one eighth of the boards.

Summary for various numbers of tables

[Balance is described as 'Excellent' if 95% or above, 'Very Good' if 90% or above, 'Good' if 80% or above.]

7 Tables

Switching one round of a 7R Mitchell gives 'Very Good' balance (90%).

8 Tables

Switching an 8R Share-&-Relay or Double Weave Mitchell gives 'Very Good' balance (90% or 93%).

9 Tables

Switching one round of a 9R Mitchell gives 'Very Good' balance (93%).

10 Tables

Switching two rounds of a 12R Extended Mitchell gives balance which is only just 'Good' (81%) but then the unswitched version is also poorly balanced because pairs meet twice. (The balance of a switched 9R Skip Mitchell is worse, only 73%.)

Another way of comparing movements is to simulate a session in which pairs of differing *known* strengths compete: a 'good' movement should produce a ranking by result that closely matches the ranking by known strength. The match can be measured by 'Spearman's Rank Correlation Coefficient' (r).[ii]

In the case of the 12R Extended Mitchell, r is 0.988 if the movement is unswitched; switched, r is 0.971. The difference appears sufficient to suggest that the unswitched movement is to be preferred.[2]

11 Tables

The 12R Hesitation Mitchell avoids having to play a 'revenge' round (which unbalances a movement). Its balance is only just 'Good' (81%). However, its r is 0.992, which looks better than that for an *unswitched* 12R Extended Mitchell (0.984).

12 Tables

Switching two rounds of a 12R Double Weave or Share-&-Relay Mitchell gives 'Good' balance (88% or 85%).

13 Tables

Switching two rounds of a 13R Mitchell gives 'Very Good' balance (93%).

14 Tables

Switching two rounds of a 13R (incomplete) Mitchell gives a balance of 79%: *not* 'good'. This confirms the advice in the EBU Manual, which advises *against* switching incomplete Mitchell movements.

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[2] The r values shown result from a specific simulation; other values might result from a different simulation.

[i] **How is 'Balance' measured?**

In a perfectly fair 1-winner movement, every pair competes with every other pair and the amounts of competition between pairs will be the same for all. If it is the same for all then in each case it is equal to the average amount of competition.

We use S to denote the total amount of competition and N to denote the number of competitions between pairs. The average is S/N.

> *In the 8T 8R Share-&-Relay Mitchell, there are 56 lots of 8 in each of the NS v NS and EW v EW quadrants of the array, and 64 lots of 0 in the NS v EW and EW v NS quadrants. S is therefore 56 x 8 x 2 = 896, and N is 56 x 2+ 64 x 2 = 240. The average S/N = 896/240 = 3.7 (approx).*

To assess how evenly the amounts of competition are spread across the array, we square each entry and sum them to form the 'sum of squares' SS. For a perfectly balanced movement, where all the amounts are S/N, the sum of squares is N x $(S/N)^2$. This is denoted by SS_{opt} ('opt' standing for 'optimal').

> *For the 8T 8R Share-&-Relay Mitchell, SS_{opt} = 240 x $(896/240)^2$ = 3345 (approx).*

For any specific movement, the actual value of SS is obtained from the array by squaring each entry and summing the results. The sum is denoted by SS_{act} ('act' standing for 'actual'). The balance of the movement is then assessed as the ratio SS_{opt}/ SS_{act}, denoted by Q and usually expressed as a percentage. If the movement is perfectly fair, SS_{act} will equal SS_{opt} and its balance Q will be 100%. In practice, movements will have Q values of less than 100%.

> *In the case of the Arrow Switched 8T 8R Share-&-Relay Mitchell, SS_{opt} is still 3345 (approx) and SS_{act} is found to be 3712 . Hence Q = 33345/3712, which as a percentage is 90%.*

The Q values can be used to distinguish between different 1-winner movements. It is possible to derive a 'Q' value for 2-winner movements but it is on a different conceptual basis and, while it can be used to distinguish between different 2-winner movements, it is less reliable for comparing a 2-winner with a 1-winner movement.

[ii] **Spearman's Rank Correlation Coefficient**

Suppose that pairs are ranked 1, 2, 3, 4, ... in order of (known) strength. The outcome of using a specific movement is a ranking by result which may produce a different ordering: say 3, 1, 2, 4, ... and so on. The differences (each denoted d) in the two rankings can be calculated (here they are 1-3, 2-1, 3-2, 4-4, ...), then squared (to give 4, 1, 1, 0, ...) and summed, to give $\sum d^2$.

Spearman's Rank Correlation Coefficient is denoted by r. If the number of pairs is n then $r = 1 - 6 \sum d^2 / n(n-1)$.

[Strictly speaking this formula applies only if there are no ties in the rankings. If there are ties then each needs to be assigned its average rank; for example, if two pairs tie for 3rd place then each is assigned a rank value of 3.5.]

Since r can be constructed for 2-winner as well as 1-winner movements on a consistent conceptual basis, it provides a way of distinguishing between movements where the balance values Q fail to provide a clear indication which is better.

However, the values of r can only be derived by simulating a bridge session, and the ranking by results and therefore the value of r will depend to a degree upon the happenstance of who actually meets whom during the course of the session.